Problem of the Week #7
11/19/2018 to 12/2/2018

For each integer $n \geq 0$, let $S(n) = n - m^2$, where $m$ is the greatest integer with $m^2 \leq n$. Define a sequence $(a_k)_{k=0}^{\infty}$ by $a_0 = A$ and $a_{k+1} = a_k + S(a_k)$ for $k \geq 0$. For what positive integers $A$ is this sequence eventually constant?

Solutions to the last problem were submitted by Suliko Bolkvadze (Georgia, the country), T.J. Gaffney (Las Vegas, NV), Rob Hill (Gambrills, Maryland), Kipp Johnson (Beaverton, OR), Tengiz Kutchava (Georgia, the country), Tom O’Neil (Central Coast of CA), Benjamin Phillabaum (Northbrook, IL), Zurab Zakaradze (Georgia, the country), and Eric Zou (San Antonio).

Solutions for this problem can be submitted to Dr. Brian Miceli at bmiceli@trinity.edu, or you can drop them off at his office, MMH 115F. People with correct solutions will be acknowledged on the next problem. For old problems, follow the “Problem of the Week” link at www.trinity.edu/bmiceli, and if you like these problems, you may be interested in the Putnam Exam. More information on the Putnam Exam can also be found at www.trinity.edu/bmiceli.