The point $P$ lies inside the equilateral triangle $ABC$ such that the distance from $P$ to $A$ is 3, the distance from $P$ to $B$ is 4, and the distance from $P$ to $C$ is 5. Find the area of triangle $ABC$.

**Solution:** The area of triangle $ABC$ is $25\sqrt{3}/4 + 9$ square units. While many problem solvers set up a system of equations by employing the law of cosines multiple times, we give a more geometric argument here.

We are given that $|PA| = 3$, $|PB| = 4$, and $|PC| = 5$. Rotate the triangle $BPA$ 60 degrees around $A$ so that the edges $AB$ and $AC$ coincide, and let $X$ be the image of the point $P$. Notice that $|PX| = 3$, $|CX| = 4$, and $|CP| = 5$. Thus, $PX \perp CX$. Also, $APX$ is an equilateral triangle of side length 3. Applying the law of cosines to $CXA$, we find that

$$|AC| = \sqrt{3^3 + 4^3 - 2 \cdot 3 \cdot 4 \cos(150\degree)} = \sqrt{25 + 12\sqrt{3}},$$

and so the area of $ABC$ is $25\sqrt{3}/4 + 9$ square units.

Solutions for this problem were submitted by Rob Hill (Gambrills, Maryland), Kipp Johnson (Beaverton, OR), Steve King (Pullman, WA), Hari Kishan (India), Thomas Plantin (TU), Surajit Rajagopal (India), Luciano Santos (Portugal), and A.Teitelman (Israel).