The children from four families, the Johnsons, Smiths, Jacksons, and Browns, are playing in the backyard. We are told that the Johnsons have the largest number of children, followed by the Smiths and then the Jacksons and then the Browns. We are also told that there are fewer than 18 children total and the product of the numbers of children in each family is equal to the house’s street number, yet this is not quite enough information. Once we are told whether or not the Browns have more than one child, we are able to deduce the number of children in each family. How many children are in each family?

Solution: The families contain 5, 4, 3, and 2 children, respectively.

We begin with the fact that there are fewer than 18 children, and that each family has a different number of children. Since factoring the house’s street number did not immediately yield the answer, that number must be able to be factored into four distinct numbers in multiple ways such that the sum of the numbers is less than 18.

We now write down various combinations of four different number which add up to less than 18 to obtain their product, and we only consider those that multiply together to equal products (otherwise we would have already solved the problem). For example, $1 \times 2 \times 3 \times 8 = 1 \times 2 \times 4 \times 6$.

Finally, we needed whether or not the Browns have more than one child, but there will be only one number in our list for which this matters, namely 120:

$$120 = 1 \times 3 \times 5 \times 8 = 1 \times 4 \times 5 \times 6 = 2 \times 3 \times 4 \times 5.$$

Since this information made it possible to deduce the number of children, it must be the case that the Browns do have more than one child.

Note: This is adapted from Problem E776 of the 1948 American Mathematical Monthly.

Solutions for this problem were submitted by Rob Hill (Gambrills, Maryland), Lincoln James (Chicago, IL), Steve King (Pullman, WA), Hari Kishan (India), Tom O’Neil (Central Coast of CA), Luciano Santos (Lisboa, Portugal), Matthias Schulte (Germany), Joe Seaborn (Indianapolis, IN), Lee (Ithaca, NY), and Waubonsee’s Math & Engineering Club (Sugar Grove, IL).