Let $M = \{1, 2, 3, \ldots, 2048\}$. If $X$ is any 15-element subset of $M$, prove that there are two disjoint subsets of $X$ whose sum of elements is the same, i.e., prove that we can find subsets $A$ and $B$ of $X$ with $A \cap B = \emptyset$ and $\sum_{a \in A} a = \sum_{b \in B} b$. If $X$ is a 12-element subset of $M$ is this result still true?

**Solution:** Initially we observe that there are $2^{15} - 1 = 32767$ possible nonempty subsets of a 15-element set $X$. Given $Y \subseteq X$, we see that $\sum_{y \in Y} y \leq \sum_{x \in X} x \leq 15 \cdot 2048 = 30720$. Since there are more possible subsets of $X$ than possible sums of the elements in the subsets of $X$, there must be two distinct subsets $E, F \subset X$ with $\sum_{n \in E} n = \sum_{n \in F} n$. Now, let $A = E - (E \cap F)$ and $B = F - (E \cap F)$, observe that $A, B$ are nonempty, as $E, F$ are distinct, and that $\sum_{a \in A} a = \sum_{b \in B} b$ as desired.

Finally, consider the case where $X = \{1, 2, 4, \ldots, 2048\} = \{2^0, 2^1, 2^2, \ldots, 2^{11}\}$, so that $|X| = 12$. If it were possible to find disjoint subsets $A, B$ of $X$ with $\sum_{a \in A} a = \sum_{b \in B} b$, we would then find two distinct binary representations for the same natural number, a contradiction. Therefore the result would not be valid in this case.

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