Alice and Bob play the following game: Bob picks any polynomial, \( P(x) \), with nonnegative integer coefficients. In each round of play, Alice chooses some integer \( n \) and Bob tells her the value of \( P(n) \). Alice then wins this game if she ever guesses Bob’s polynomial. To make things interesting, Bob puts $500 on the table, removing $100 after each specific value of \( P \) he provides to Alice. Show that Alice can employ a strategy to always walk away with at least $300.

(This will be the last problem of the school year, and we’ll start up again in August.)

Thanks to all of those who submitted solutions to last week’s problem. The complete list of people who submitted solutions to last week’s problem can be found on the posted solution.

Solutions for this problem can be submitted to Dr. Brian Miceli at bmiceli@trinity.edu, or you can drop them off at his office, MMH 115F. People with correct solutions will be acknowledged on the next problem. For old problems, follow the “Problem of the Week” link at www.trinity.edu/bmiceli, and if you like these problems, you may be interested in the Putnam Exam. More information on the Putnam Exam can also be found at www.trinity.edu/bmiceli.