Suppose we are given 64 points in the plane which are positioned so that 2001, but no more, distinct lines can be drawn through pairs of points. Prove that at least four of the points are collinear.

Solution: We notice that there are \( \binom{64}{2} = \frac{64 \cdot 63}{2} = 2016 \) distinct pairs of points to choose from, but since there are exactly 2001 lines, at least some three points are collinear.

Suppose that no four points are collinear, that is, every line either connects only two points or exactly three points. We now notice that if a line contains exactly three points, then this line should be counted once, but if those three points were not collinear, then there would be three distinct lines connecting them. Accordingly, if \( k \) is the number of lines connecting three collinear points, then the total number of lines connecting all 64 points is \( 2016 - 2k \). Since this value is always even, there cannot be exactly 2001 lines in this scenario, and so there exist at least four collinear points.

(Note: This is Problem 79 is from *The Wohascum County Problem Book* by Gilbert et al., and it caught my eye because it involved a 2016!)

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