Problem of the Week #10
1/11/2016 to 1/24/2016

Given that \( \lim_{x \to \infty} (f(x) + f'(x)) = 0 \), prove that \( \lim_{x \to \infty} f(x) = \lim_{x \to \infty} f'(x) = 0 \).

Solution: Suppose that \( \lim_{x \to \infty} f(x) = L \). If \( L = 0 \) we are done, and otherwise, as \( \lim_{x \to \infty} e^x = \infty \), l’Hospital’s Rule gives that,

\[
\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{f(x)e^x}{e^x} = \lim_{x \to \infty} \frac{(f(x)e^x)'}{e^x} = \lim_{x \to \infty} \frac{f(x)e^x + f'(x)e^x}{e^x} = \lim_{x \to \infty} (f(x) + f'(x)) = 0.
\]

Thus, \( \lim_{x \to \infty} f(x) = 0 \). Moreover, \( 0 = \lim_{x \to \infty} (f(x) + f'(x)) = \lim_{x \to \infty} f'(x) \), and we are done.

The only solution for this problem was submitted by Hari Kishan (India).